Frequency Recognition of a Free Damped Vibration Signal Based on Improved Elias Aboutanios Interpolation Method

Feng Peng[1,2, a], Chang Deng [1,2, b], Jiaqi Wen [1,2, c] and Fuhai Guan [1,2, d]

[1] State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering Nanjing 210029, China;

[2] Nanjing Hydraulic Research Institute, Nanjing 210029, China.

[a] fpeng@nhri.cn, [b] cdeng@nhri.com, [c] jqwen@nhri.cn, [d] fhguan@nhri.cn

Abstract. The free damped vibration signal of engineering structures is an energy-limited signal, and the effective signal length is limited by the damped factor. The classical periodic graph method is affected by signal damped coefficient and noise, and it is difficult to obtain satisfactory recognition accuracy. This article proposes the IEAI method for frequency estimation of freely damped vibration signal, considering the influence of initial phase. By combining discrete Fourier transform with complex operations, the signal is iteratively interpolated to improve the accuracy of frequency estimation .The simulation results show that the mean error of the IEAI method is 1.3%~26.5% of the traditional Periodogram method, A&M method and Welch method, and the Root-mean-square deviation is 7.4%~40.3% of the latter three methods under the condition of high signal-to-noise ratio; In the case of low signal-to-noise ratio, the mean error of the IEAI method is 16.2%~43.0% of the latter three, and the Root-mean-square deviation is 20.8%~81.8% of the latter three. The experimental results of cantilever beam damage identification show that the IEAI method is more sensitive to small structural damages and can be used for the identification and diagnosis of small structural damages.

Keywords: structural vibration signal; frequency estimation; improved interpolation; simulation analysis; damage Identification.

1. Introduction

In the field of engineering structural safety monitoring and damage identification, the free damped vibration signal is commonly used for modal parameter identification. Modal parameters can reflect the dynamic characteristics of the structure. Vibration based damage identification methods diagnose the structural state through modal parameters[1, 2, 3, 4], and structural frequency is an important identification parameter. The actual structure is in an uncertain environment, and frequency identification is greatly affected by environmental factors, requiring high estimation accuracy. Relevant experiments show that damage identification based on frequency parameter changes is difficult to identify small structural damages[5].

The frequency identification methods based on dynamic test data mainly include periodogram method[6], maximum likelihood method[7], autocorrelation method[8], interpolation method[9], etc. Among them, periodogram method is a widely used method, which is simple, has clear physical concepts, and high computational efficiency. However, the periodogram method obtains discrete frequency values, and frequency estimation is accurate only when the signal frequency is an integer multiple of Δf . When the signal frequency does not coincide with the discrete frequency, the periodogram method does not meet the consistency estimation condition[10]. Therefore, domestic and foreign scholars propose improved methods based on the classical periodogram method, such as the ratio method proposed by Rife and Vincent[11]; Aboutanios propose interpolation method[9] and A&M method[12], which to some extent reduce the influence of discrete frequency values through quadratic estimation; The Bartlet[13] method and Welch[14] method improve accuracy by adding a windowed average. These methods have high accuracy in frequency estimation of sinusoidal signal, but they do not consider the impact of the damping coefficient of the free damped vibration signal on frequency estimation. Therefore, the results of frequency estimation of attenuated signal are not ideal.

In summary, although the traditional power spectrum method based on the periodic graph method is simple to calculate, it cannot balance accuracy and stability when identifying the frequency of free damped vibration. This article proposes an improved interpolation method (IEAI method) based on the interpolation method and combined with the characteristics of free damped vibration signal, which retains the fast calculation characteristics of the periodic graph method and improves the frequency estimation accuracy of free damped vibration signal. The specific works are as follows:

(1) Compare the estimation results of free damped vibration signal containing Gaussian white noise using periodogram method, conventional interpolation method, and A&M method;

(2) Propose the IEAI method, which combines the characteristics of free damped signal to effectively eliminate the influence of initial phase on frequency estimation, thereby obtaining accurate frequency estimation values;

(3) Simulate and verify the accuracy and stability of the improved interpolation method, and compare the accuracy of IEAI method with the periodogram method, A&M method, and Welch method under different signal-to-noise ratios;

(4) Introducing measured vibration data of cantilever beams to compare the frequency changes before and after damage, further verifying the identification accuracy of the IEAI method.

2. Comparison of Three Spectral Estimation Methods

According to literature, the periodogram method has good stability under high signal-to-noise ratio conditions(SNR). The interpolation method has good accuracy and strong robustness in frequency estimation, and is not easily affected by discrete frequency values. The A&M method has good accuracy and stability, but is sensitive to noise. Therefore, the following is a comparative research of these three spectral estimation methods.

2.1 Classic Periodogram Method

The periodogram method can be traced back to Schuster's work[6] in the late 19th century. Schuster propose the concept of periodogram for the observation sequence$\{x_k, k = 1,2, \ldots, N\}$, and its expression is:

$$\boldsymbol{S}(\boldsymbol{\omega}) = \frac{1}{\boldsymbol{N}}\left|\sum_{\boldsymbol{k}=1}^{\boldsymbol{N}} \boldsymbol{x_n} \boldsymbol{e}^{-\boldsymbol{\omega N j}}\right| \tag{1}$$

By using the above equation, we can find the "hidden periodicity" of the observed data. The time series is formed by the superposition of a sinusoidal signal with an angular frequency of $\boldsymbol{\omega_c}$ and noise, and a peak appears at point $\boldsymbol{\omega} = \boldsymbol{\omega_c}$ in the periodic graph. In the study of water flow pressure pulsation and its induced hydraulic structure vibration cavitation and erosion, the periodogram method is a classic method for estimating the power spectral density of water flow signal pressure pulsation, flow induced vibration, etc[15].

However, this algorithm has the following drawbacks: firstly, it estimates the result of an infinite sampling sequence from a finite length of data segment, which inevitably leads to energy leakage[16]; Secondly, since the computer uses discrete Fourier transform to obtain a discrete spectrum, the adjacent spectral lines of the spectrum have a spacing of $\Delta\boldsymbol{\omega} = 2\boldsymbol{\pi}/\mathrm{N}$, and the main lobe width of the spectrum is $\Delta\boldsymbol{\omega} = 4\boldsymbol{\pi}/\mathrm{N}$. When the signal frequency $2\boldsymbol{\pi f}_0$ is not an integer multiple of $\Delta\boldsymbol{\omega}$, the maximum value after Fourier transform is located between these two sampling points, which limits the accuracy of the classical periodogram method.

2.2 Conventional Interpolation Method

the Elias Aboutanios interpolation method [9], whose frequency estimation formula is:

$$f_0 = \frac{f_s}{N}(m + \delta) \tag{2}$$

In the formula, $\frac{f_s}{N}$ is the frequency resolution, m is the maximum index value of the Discrete Fourier Transform (DFT) [17], $\delta \in [-0.5, 0.5]$ is the frequency offset value, and the superposition of δ represents the expression:

$$\mathrm{h}(\delta) = \frac{1}{2}\left(\frac{|X_{0.5}|-|X_{-0.5}|}{|X_{0.5}|+|X_{-0.5}|}\right) \tag{3}$$

In the formula, $X_{\pm 0.5}$ represents the interpolation on both sides of the maximum spectral line, and its expression is:

$$X_{\pm p} = \sum_{k=1}^{N} x_k e^{-2\pi k \frac{m \pm p}{N} j} \tag{4}$$

By iterating multiple times according to formulas (2) to (4), frequency estimation of sinusoidal signal can be achieved. Unlike the EA interpolation method, the Q-interpolation method[18] first interpolates between two spectral lines, and then uses the ratio of the interpolated spectral lines to obtain the frequency estimation result, making the algorithm more convenient. However, for free damped vibration signal with low SNR, the interpolation method may cause reverse interpolation due to the influence of damped coefficient and noise, which often leads to an increase in frequency estimation error.

2.3 A&M Method

In order to extend the interpolation method from the undamped case to the damped case, Aboutanios,Mulgrew et al. conducted in-depth research on the parameter identification algorithm for damped vibration signal and proposed the A&M method[12]. The damped vibration signal model is considered as:

$$X(k) = Ae^{(-\xi + 2j\pi f)} \tag{5}$$

The frequency estimation formula for the A&M method is:

$$f_0 = \frac{f_s}{N}(m + \delta) \tag{6}$$

$$z = 1/\left(\cos\left(\frac{\pi}{N}\right) + 2j\, h \sin\left(\frac{\pi}{N}\right)\right) \tag{7}$$

$$\delta = \frac{N}{2\pi}\angle z \tag{8}$$

In the formula, h can be obtained from equation (3). According to formulas (5) to (8), frequency estimation of damped signal can be achieved. The calculation is simple, and the estimation accuracy for high SNR damped signal is high, with good robustness. However, due to the influence of damped coefficient and noise, the stability of recognition accuracy is poor when there is a significant difference in environmental SNR.

2.4 Simulation Comparison

In order to compare the estimation results of free damped vibration signal containing Gaussian white noise using periodogram method, conventional interpolation method, and A&M method. Select 100 sets of free damped vibration signal containing Gaussian white noise for experiments, and the signal model is shown in equation (9). SNR is defined as $SNR = 10lg\,(P_{signal}/P_{noise})$, which performs frequency estimation when the phase, amplitude, and frequency parameters are all unknown. The discrete model of a freely damped vibration signal containing Gaussian white noise [19] is:

$$\begin{aligned} y &= Ae^{\frac{-\xi 2\pi f_0 k}{f_s}} \cos\left(\frac{f_0}{f_s} 2\pi k + \varphi\right) + z(k) \\ &= \frac{A}{2} e^{\frac{-\xi 2\pi f_0 k}{f_s} + \varphi j} \left(e^{\frac{2\pi f_0 k}{f_s} j} + e^{-\frac{2\pi f_0 k}{f_s} j}\right) + z(k) \qquad (0 \le k \le N) \end{aligned} \tag{9}$$

Among them, A, f_0, ξ and φ Represent the amplitude, frequency, damping, and initial phase of the signal respectively, with a sampling interval of f_s. N is the number of samples, and z (k) is Gaussian white noise. Sampling interval $T_s = 0.001\ s$, sampling points $N = 10000$. Simulated signal frequency $f = 9.875\text{Hz}$, damping ratio $\xi = 0.05$, initial phase $\varphi = 1.075$ Conduct simulation tests on simulated damped signal. Figure 1 shows the waveform of a free damped vibration signal with noise (SNR=0).

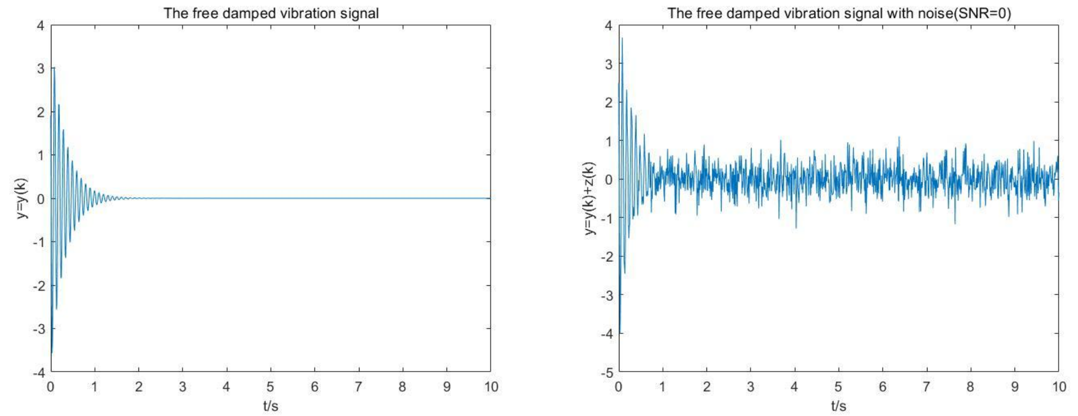

Fig. 1 Waveform graph of the free damped vibration signal

Frequency identification simulation experiments are conducted on free damped vibration signals with SNR ranging from 0 to 30 using periodogram method, EA interpolation method (iteration number Q=10), Q interpolation method, and A&M method, respectively. The calculation results are shown in Table 1 and Figure 2.

Table 1. Four Estimation Methods comparing

Method	Error	SNR						
		30	25	20	15	10	5	0
Periodogram	Mean error(%)	0.24	0.24	0.18	0.05	0.02	0.01	0.16
	RMSE	0.02	0.02	0.03	0.04	0.06	0.06	0.08
EA Interpolation	Mean error(%)	1.14	1.14	1.08	0.96	0.90	0.90	0.78
	RMSE	0.11	0.11	0.11	0.10	0.11	0.11	0.11
Q Interpolation	Mean error(%)	0.65	0.86	0.72	0.68	0.75	0.76	0.56
	RMSE	0.15	0.14	0.13	0.13	0.13	0.14	0.15
A&M	Mean error(%)	0.01	0.01	0.01	0.01	0.10	0.10	0.13
	RMSE	0.00	0.01	0.01	0.03	0.09	0.09	0.21

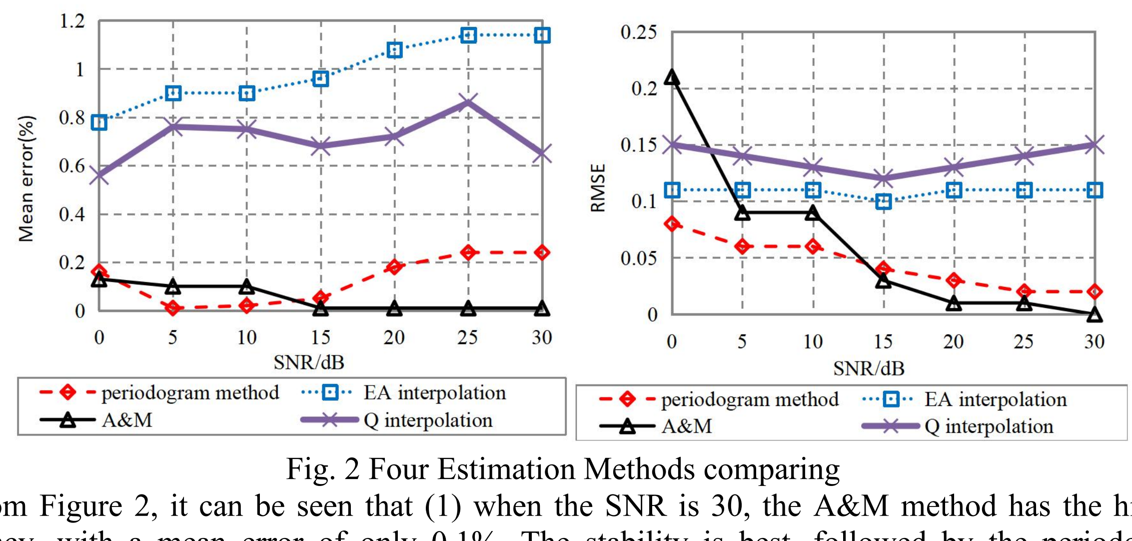

Fig. 2 Four Estimation Methods comparing

From Figure 2, it can be seen that (1) when the SNR is 30, the A&M method has the highest accuracy, with a mean error of only 0.1%. The stability is best, followed by the periodogram method with lower accuracy and poorer stability. The EA interpolation method has the worst accuracy, followed by the Q interpolation method, while the stability Q interpolation method is slightly better than the EA interpolation method; (2) When the SNR is 15, 20, and 25, the A&M method still has the best estimation accuracy and stability, with a mean error of around 0.3%, followed by the periodogram method. The stability and accuracy of EA interpolation and Q interpolation methods are still not ideal; (3) When the SNR is 5 and 10, the periodogram method outperforms the A&M method in both accuracy and stability, with a mean error below 0.5%, while the interpolation method has the worst accuracy and stability; (4) When the SNR is 0, although the periodogram method is still better than the A&M method in terms of stability, the estimation accuracy of the former is relatively poor compared to the latter, while the interpolation method has the worst accuracy and stability.

The above simulation results show that traditional power spectrum estimation methods are difficult to achieve satisfactory recognition accuracy and stability. Among them, the periodogram method has poor accuracy in high SNR, while the A&M method has a sharp decrease in stability and weak robustness in low SNR, and can only accurately identify frequencies in specific environments. Therefore, in order to achieve real-time and accurate frequency estimation of short-term wide and strong interference signal, it is necessary to improve the interpolation based on the classical periodogram method. Elias Aboutanios propose a frequency estimation formula for sine signal interpolation method. However, when this algorithm is applied to freely damped signal, its recognition accuracy is poor. In order to improve the frequency identification accuracy of free damped vibration signal, an improved algorithm is proposed based on interpolation method.

3. IEAI Method

A frequency estimation improved method (IEAI, Improved Elias Aboutanios Interpolation) is proposed by combining the characteristics of free damped vibration signal and the Aboutanios interpolation method. The specific process is shown in Figure 3. The main idea of the IEAI method is to obtain the initial phase and rough estimated frequency of the signal through the discrete Fourier transform and complex operation of the freely decaying vibration signal in the rough estimation stage, and then remove the initial phase deviation in the input signal to obtain the corrected signal. In the precision estimation stage, interpolation and complex operations are used to estimate the remaining frequency deviation through multiple iterations.

Firstly, perform Discrete Fourier Transform (DFT) [10] on the free damped vibration signal, search for the index $\hat{m}$ that returns the maximum amplitude of the power spectrum, and then calculate the DFT coefficients on both sides of the peak for interpolation estimation of frequency. In the absence of noise in an ideal signal, assuming that m is the index of the true maximum value, i.e. $\hat{m} = m$, the frequency of the signal can be written as

$$f_0 = \frac{f_s}{N}(m + \delta) \tag{10}$$

In the formula, δ is the residual in the interval $[-0.5, 0.5]$, considering the DFT transformation:

$$X_l = \sum_{n=0}^{N-1} y(n) e^{-j\frac{m+l}{N}2k\pi} + w(k) \left(k = 0,1,\cdots, N-1; |l| < m\right) \tag{11}$$

Where $w(k)$ is the FFT transformation of $z(k)$. Ignoring $w(k)$, substitute the free damped vibration signal equation (9) into equation (11) and simplify it to:

$$X_l = \frac{A}{2} e^{j\varphi} \left(\frac{1 - e^{2\pi(-\xi(m+\delta)+j(\delta-l))\frac{N-1}{N}}}{1 - e^{2\pi(-\xi(m+\delta)+j(\delta-l))\frac{1}{N}}} + \frac{1 - e^{2\pi(-\xi(m+\delta)-j(2m+\delta+l))\frac{N-1}{N}}}{1 - e^{2\pi(-\xi(m+\delta)-j(2m+\delta+l))\frac{1}{N}}} \right) \tag{12}$$

Considering $\left(-\xi(m+\delta) + j(\delta - l)\right) \ll N$ and $\frac{N-1}{N} \approx 1$, simplifying equation (12) yields:

$$X_l \approx -\frac{AN}{4\pi} e^{j\varphi} \frac{1}{\left(-\xi(m+\delta)+j(\delta-l)\right)} \tag{13}$$

Using the Euler formula, expand equation (13) to obtain:

$$-\frac{AN}{4\pi} \frac{\cos\varphi + j sin\varphi}{\left(-\xi(m+\delta) + j(\delta - l)\right)}$$

$$= -\frac{AN}{4\pi} \frac{1}{\left(-\xi(m+\delta)cos\varphi + (\delta - l)sin\varphi\right) + j\left(\xi(m+\delta)sin\varphi + cos\varphi(\delta - l)\right)}$$

Further use real and imag functions to separate the real and imaginary parts in the denominator of the above equation to obtain:

$$\hat{\varphi} \approx arctan(\frac{real\left(\frac{1}{X_l}\right)-real\left(\frac{1}{X_{-l}}\right)}{iamg\left(\frac{1}{X_l}\right)-imag\left(\frac{1}{X_{-l}}\right)}) \tag{14}$$

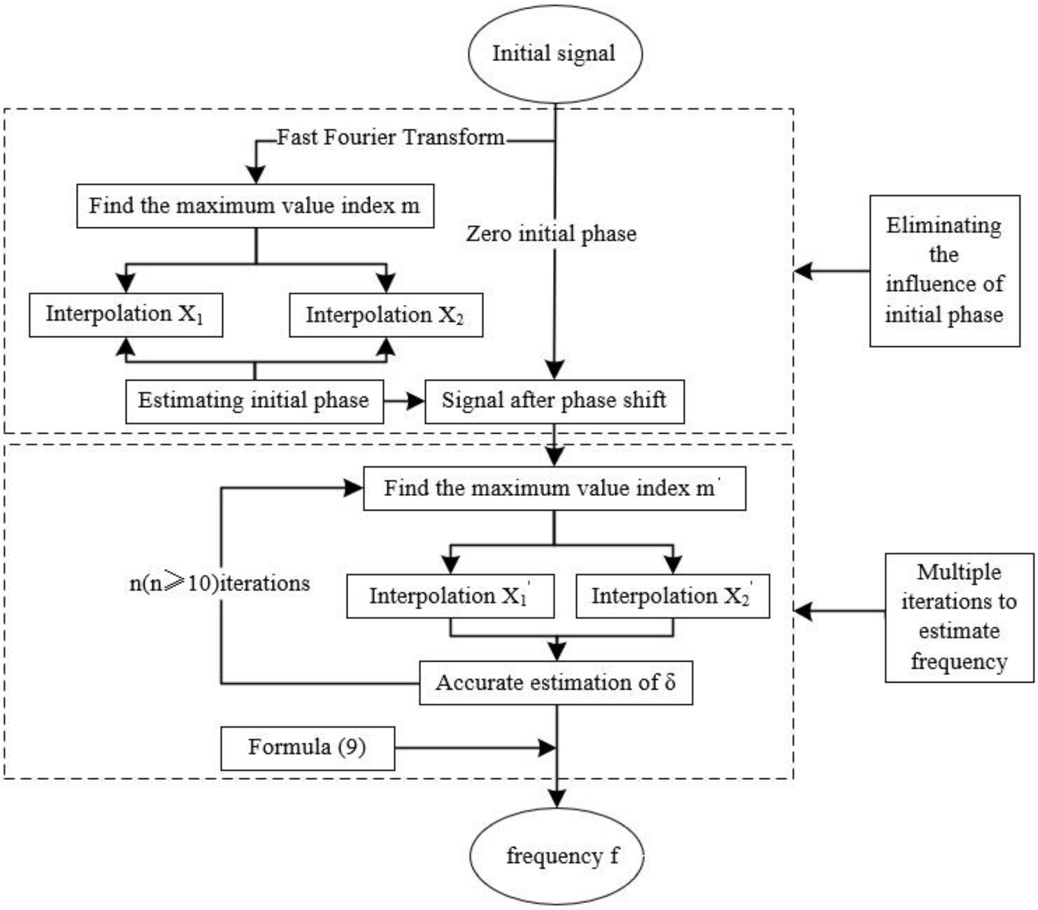

Fig. 3 The flowchart of IEAI method

Finally, by separating the imaginary part of A in equation (13), it can be obtained that：

$$X_p^{'} = \frac{1}{imag\left(\frac{\frac{1}{X_p}}{e^{j\hat{\varphi}}}\right)} = -\frac{AN}{4\pi}\frac{1}{((\delta-P))}(p = \pm 0.5) \tag{15}$$

Due to $|\delta| \leq 0.5$, it can be concluded that:

$$\mathrm{h}(\delta) = \frac{1}{2}\left(\frac{|X_{0.5}^{'}|-|X_{-0.5}^{'}|}{|X_{0.5}^{'}|+|X_{-0.5}^{'}|}\right) = \frac{1}{2}\frac{\left|\frac{1}{(\delta-0.5)}\right|-\left|\frac{1}{(\delta+0.5)}\right|}{\left|\frac{1}{(\delta-0.5)}\right|+\left|\frac{1}{(\delta+0.5)}\right|} = \delta \tag{16}$$

In summary, $\delta = h(\delta)$ can be used as an estimate of δ.

4. Verification I: Simulation Analysis

In order to verify the recognition accuracy and stability of the IEAI method (iteration number Q=10) and the Welch method (Hamming window, observation length taking 2048 time steps, overlapping 512 time steps between segments), 100 sets of free damped vibration signals containing Gaussian white noise were selected for experiments. The signal model is shown in equation (9), and the calculation results are shown in Table 2 and Figure 4.

Table 2. Four Estimation Methods comparing

Method	Error	SNR						
		30	25	20	15	10	5	0
Periodogram	Mean error(%)	0.24	0.24	0.18	0.05	0.02	0.01	0.16
	RMSE	0.02	0.02	0.03	0.04	0.06	0.06	0.08
A&M	Mean error(%)	0.01	0.01	0.01	0.01	0.10	0.10	0.13

	RMSE	0.00	0.01	0.01	0.03	0.09	0.09	0.21
IEAI	Mean error(%)	0.24	0.24	0.24	0.24	0.24	0.17	0.06
	RMSE	0.02	0.02	0.02	0.02	0.02	0.03	0.04
Welch	Mean error(%)	0.00	0.00	0.01	0.01	0.00	0.01	0.03
	RMSE	0.00	0.00	0.01	0.01	0.02	0.04	0.06

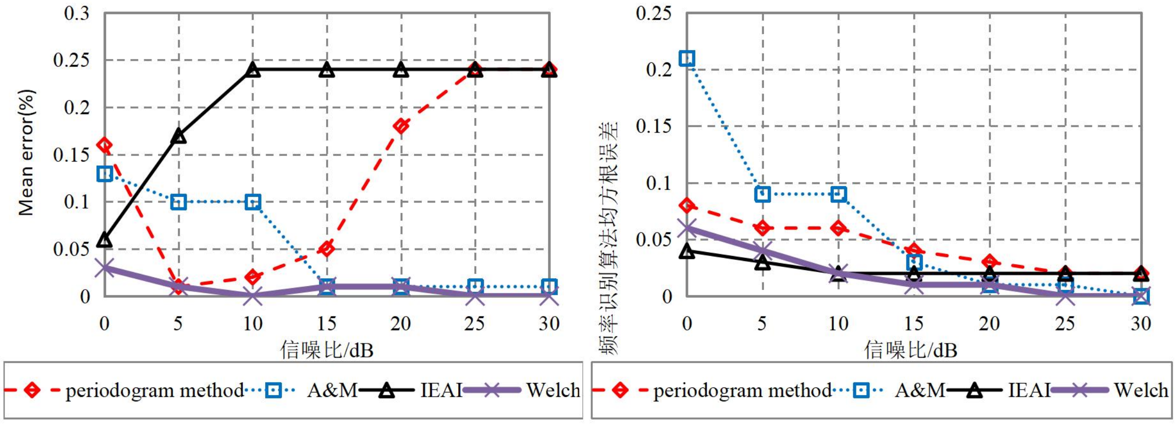

Fig. 4 Four Estimation Methods comparing

The estimation results of each method under different SNR are shown in Figure 4: (1) When the SNR is 30, the improved interpolation method has the highest accuracy, with a mean error of only 0.1%, and the stability is the best. The A&M method and periodic graph method have the second highest accuracy, and the stability is poor. Although the Welch method has good stability, the accuracy is not ideal; (2) When the SNR is 15, 20, and 25, the improved interpolation method still has the best estimation accuracy and stability, with a mean error of around 0.1%. The A&M method and periodic graph method have the second highest accuracy, especially the periodic graph method. The accuracy improves significantly with the decrease of SNR. The Welch method still has good stability, but the accuracy is not ideal; (4) When the SNR is 0, 5, and 10, the stability of the Welch method surpasses that of the improved interpolation method, while IEAI method still has the best estimation accuracy. The accuracy and stability of the A&M method and periodic graph method have decreased.

From this, it can be seen that the Welch method, due to its inability to perform sufficient data segmentation, will limit its performance. When the SNR is high, the mean error of frequency recognition is four times that of low SNR ratio, and the recognition accuracy is lower than that of the A&M method; The A&M method has the characteristics of high recognition accuracy and algorithm stability in high SNR situations. However, in low SNR, the mean error of frequency recognition is 10 times that of high signal-to-noise ratio situations, and the RMSE of frequency recognition can even reach more than 20 times that of high SNR; The IEAI effectively solves the problem of poor accuracy and weak robustness of A&M method in low SNR. It can maintain high recognition accuracy and good stability in both low and high SNR.

5. Verification II: Actual Measurement Analysis

A certain cantilever beam (Figure 5), weighing 6.230kg, with a total length of 3.07m and a cantilever length of 1.87m, is tested to determine if there is any damage to the cantilever beam. This test has 5000 sampling points, a sampling time of 5 seconds, and a sampling rate of 1000Hz. All observed data were converted into free damped vibration signal through analog-to-digital conversion, and vibration images are drawn. The detection schematic diagram in time domain and power spectrum are shown in Figure 6.

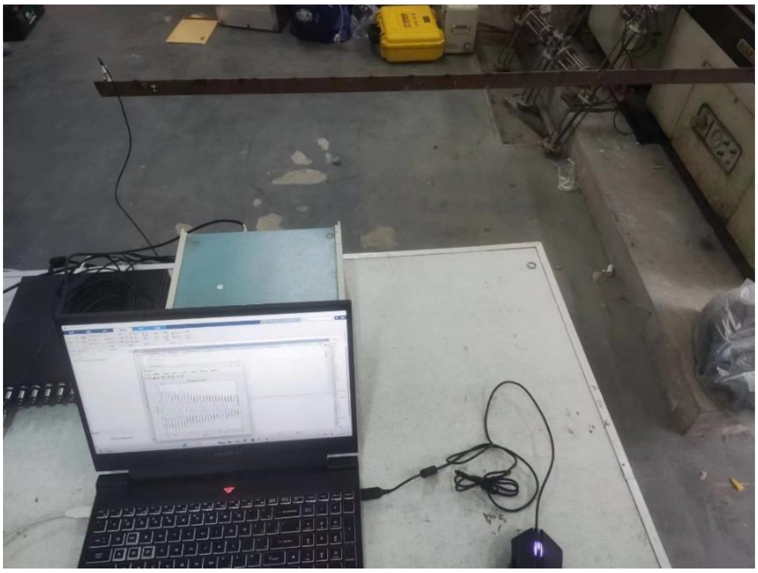

Fig. 5 Cantilever beam model and vibration testing system

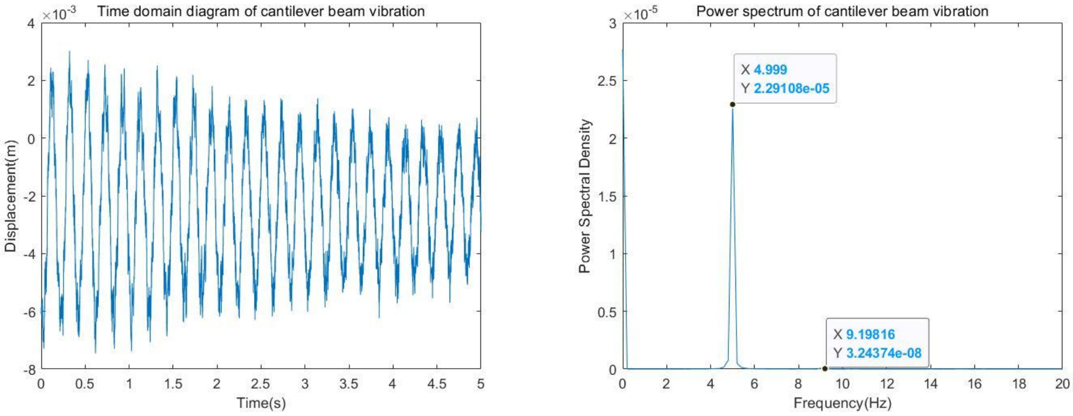

Fig. 6 Time domain and power spectrum diagrams of cantilever beam vibration

The cantilever beam model and vibration testing system artificially created small damages near the support end of the cantilever beam (Figure 7), with a width of about 0.5mm and a depth of 1mm. The frequency estimation changes of various frequency identification methods before and after the damage were compared, as shown in Table 3.

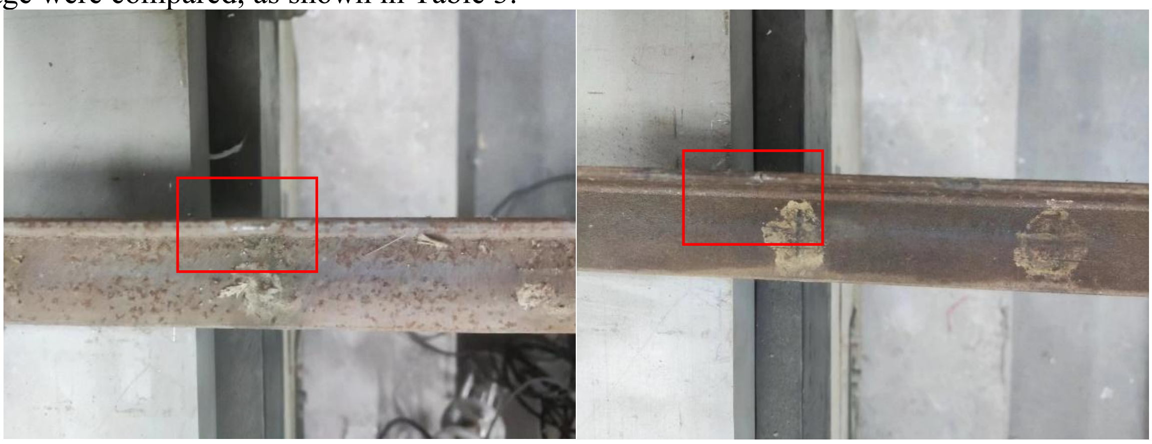

Fig. 7 Comparison of cantilever beam damage before and after

Table 3. Six estimation methods comparing

Method	Before Damage		After Damage	
	First order	Second order	First order	Second order

	frequency	frequency	frequency	frequency
Periodogram	5	9.2	5	9.2
EA Interpolation	4.98	9.1	4.98	9.1
Q Interpolation	4.99	9.12	4.99	9.12
Welch	5	9.2	5	9.2
A&M	4.98	9.1	4.98	9.1
IEAI	4.98	9.1	4.97	9.08

From the table, it can be seen that the first frequency variation identified by the classical periodogram method, EA interpolation method, Q interpolation method, Welch method, and A&M method is weak, while the second frequency variation is almost unchanged. The IEAI method can identify the frequency variation caused by minor damage, with the first and second frequencies before damage being 4.98 and 9.10, and after damage being 4.97 and 9.08, respectively. Therefore, the IEAI method can identify the frequency changes caused by minor damage and can be used to identify structural minor damage.

6. Summary

When the classical sine signal frequency identification method is applied to identify the frequency parameters of freely damped vibration signal, it is often difficult to achieve satisfactory identification accuracy due to the influence of damped coefficient and noise. Therefore, in order to achieve real-time and accurate frequency estimation of free damepd vibration signal, this paper proposes the IEAI method based on interpolation method and free damped signal characteristics. The research conclusion is as follows:

(1) When the A&M method is applied to frequency identification of free damped vibration signal, the algorithm is very sensitive to noise. When the SNR of the signal is low, the identification error increases sharply, and the algorithm's robustness is weak. The Welch periodogram algorithm has good noise resistance, but when the SNR is high, the frequency recognition accuracy is relatively poor.

(2) By combining discrete Fourier transform with complex operations, the influence of initial phase is eliminated, and iterative interpolation is performed on the corrected signal to propose the IEAI method for frequency estimation of free damped vibration signal.

(3) The simulation results show that when the SNR is low, the recognition accuracy of IEAI method is much higher than A&M method and Welch method. In high SNR, the recognition accuracy of IEAI method is also better than A&M method and Welch method. Therefore, IEAI method has the characteristics of high recognition accuracy and stronger robustness.

(4) The analysis of actual test results shows that compared with traditional power spectrum estimation methods, IEAI method can effectively suppress environmental noise, identify frequency changes before and after minor structural damage, and can be used to identify minor structural damage.

Acknowledgements

Funding: This work was supported by the National Key Research and Development Plan (Chinese, Grant No. 2021YFC3090103).

References

[1] Favarelli E, GiorgettiA. Machine learning for automatic processing of modal analysis in damage detection of bridges[J].IEEE Transactions on Instrumentation and Measurement,2021,70: 113.

[2] Wang ZC, Ge B, Ren WX, et al. Discrete analytical mode decomposition with automatic bisecting frequency selection for structural dynamic response analysis and modal identification[J]. Journal of Sound and Vibration,2020,484: 115520.

[3] WANG Hui, WANG Le,TIAN Runze. Structural damage detection using correlation functions of time domain vibration responses and data fusion[J].Engineering Mechanics, 2020,37(09):30-37+111.

[4] LI Huimin, DONG Meimei, XIONG Xiong, et al. State-of-the-art review of vibration-based damage identification framework for structures[J].Building Structure, 2021,51(04):45-50+38.

[5] Zong Zhouhong, Zhang Kun, Liao Yuchen, et al. Modal Frequency Identification of Cable-stayed Bridges Considering Uncertainties of Operational Environmental Factors[J]. China Journal Of Highway And Transport, 2019,32(11):40-50.

[6] SCHUSTER A. On hidden periodicities[J].Terrestrial Magnetism,1898(3) : 13-41.

[7] Guo Xianbin Guo Pan.et al. Performance Analysis of Maximum Likelihood Spectral Estimator Compared with PM Estimator [J]. Chinese Laser, 2016, 43 (03): 236-243.

[8] HU Lei, SHEN Jihong. Modified high-lag autocorrelation estimation method for frequency estimation of sinusoidal signal [J]. Journal of Harbin Engineering University, 2016, 37 (04): 579-584. (in Chinese).

[9] Elias Aboutanios, Bernard Mulgrew. Iterative Frequency Estimation by Interpolation on Fourier Coefficients[J].IEEE Transactions on Signal Processing: A publication of the IEEE Signal Processing Society,2005,53(4).

[10] Wang Hongwei, Zhao Guoqing. Improved Rife Algorithm for Frequency Estimation of Sinuoid Wave[J].Jourunal of Signal Processing, 2010,26(10):1573-1576.

[11] Rife C D, Vincent A G. Use of the Discrete Fourier Transform in the Measurement of Frequencies and Levels of Tones[J]. Bell System Technical Journal,1970,49(2).

[12] Elias Aboutanios. Estimation of the Frequency and Decay Factor of a Decaying Exponential in Noise[J]. IEEE Transactions on Signal Processing: A publication of the IEEE Signal Processing Society,2010,58(2).

[13] Song Ning, GUAN Hua. Classical Power Spectrum Density Estimation and Its Simulation[J]. Modern Electronics Technique, 2008(11):159-161+164.

[14] YANG Jing, CHENG Nai ping.et al. Performance Analysis of Welch Algorithm in Weak Signal Detection[J]. Computer Simulation, 2020,37(05):235-240.

[15] Luo Shaoze, Yan Shiwu. Maximum entropy spectrum estimation program for water flow signals and its application[J]. Hydro-Science and Engineering, 1992(03):245-254.

[16] Harris F. On the use of windows for harmonic analysis with the discrete Fourier transform[J]. Proceedings of the IEEE,1978,66(1).

[17] L.C. Palmer. Coarse frequency estimation using the discrete Fourier transform [J]. IEEE Trans Inform Theory,1974,IT20(1):pp104-109.

[18] B. G. Quinn. Estimation of frequency, amplitude, and phase from the DFT of a time series[J]. IEEE trans. on signal process., Mar. 1997, 45(3):814-817.

[19] SUN Wei, QI Fei, HAN Qing-kai. Estimating system damping for a hard coating composite structure based on envelope of a free damped vibration signal[J]. Jourunal of Vibration and Shock,2013,32(12):50-54.