

On the solution of seven-precision-point path synthesis of planar four-bar linkages based on the solution region methodology

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Abstract. This paper makes further research on the application of the solution region synthesis methodology (SRSM) in path synthesis of planar four-bar linkages. Detailed process of establishing the solution region for seven precision-point path synthesis is illustrated. In order to obtain the solution region where one point corresponds to one linkage and the feasible solution region where linkages distributes different zones according to curve types, this paper compares four different solution region schemes and finally establishes the solution region with B_1 as the reference. Compared with the eight-point case, the range of the solution region in this paper is larger and the statistics on the feasible solutions is more than twice of that in the case with eight points. Moreover, the feasible solutions obtained in this case have more curve types.

Keywords: four-bar linkages, path synthesis, solution region methodology.

1. Introduction

Path synthesis of planar four-bar linkages is an important component of dimensional synthesis, which can be performed by analytical or optimization methods. Analytical methods can obtain linkages which can pass through the given points exactly, but it is often accompanied by solving large highly nonlinear polynomial equations. These equations are difficult to be solved by the traditional analytical methods, that is, to solve the polynomial of high degree with one variable obtained through elimination algorithms. The development of numerical continuous method [1,2] greatly promotes the research of precise point path synthesis of linkages. Wampler et al. [1] came to the conclusion that complete solution of the nine-precision-point path synthesis problem has 4326 isolated nondegenerate solutions. But this method will produce a large number of iterative paths that do not converge in the process of solving large polynomials. In order to reduce the number of divergent paths, References [3-8] presented some novel path search techniques, which can efficiently reduce the divergent paths. The computational efficiency was significantly improved by the application of the regeneration homotopy method [3]. Using these advanced techniques, Plecnik et al. [5, 9-15] studied the path synthesis and timed curve generator synthesis of four-bar and six-bar linkages.

Since it is not limited by the number of given points, scholars have carried out extensive research on the optimization methods for path synthesis. Generally, optimization methods are to give the optimization objectives, and then use certain optimization algorithms to acquire the optimal solution. At present, the optimization algorithms applied to the path synthesis of linkages mainly conclude Evolution Algorithm[16-18], Teaching-Learning-Based-Optimization[19], Krill Herd Algorithm[20] and Circular Proximity Function[21]. In order to make the design process of linkages more intuitive, Torres-Moreno[22] developed an open source software for path synthesis of four-bar linkages based on optimization algorithms. Although path synthesis using optimization methods is not limited by the number of given points, it is not applicable in some accurately. Moreover, usually only one optimal linkage solution can be produced each time.

In addition to the optimization methods above, there is another type of optimization method, which is called numerical atlas method. The major work of numerical atlas method is to extract the feature information of the coupler curves using different techniques, and establish the database. At present, popular techniques mainly use Fourier series[23-27], harmonic characteristic parameters[28], wavelet feature parameters[29, 30] to extract the feature information of the coupler curves. Similarly, numerical atlas methods can only get one or several usable linkages once, which cannot pass through the given points exactly.

For nine-point path synthesis, only several linkages or even no linkages without defects can be obtained. Although a certain number of useful linkages are obtained in the case for eight-points, curve types of the useful solutions are with poor diversity. Since just one of the eight unknowns is given, the establishment of the solution region is relatively simple. Because two unknowns must be given for seven-point synthesis, more linkages can be obtained and the establishment of the solution region becomes more complex. And some new problems appear in the process of establishing feasible solution region, such as the situation that one point corresponds to multiple four-bar linkages in feasible solution region. Therefore, this paper makes further study to find the solution region type that is most appropriate for seven-precision-point path synthesis. In a certain given range of A_0 , after imposing design requirements and eliminating linkages with defects, the feasible solution region is obtained, in which 476 usable linkages are obtained and 426 of them are crank-rocker linkages.

2. Solution Region Establishment

2.1 Formula For Synthesis

Figure 1 is the schematic of a four-bar linkage from position 1 to position i . $A_0(a_{0x}, a_{0y})^T$ and $B_0(b_{0x}, b_{0y})^T$ are vectors of fixed pivots. $A_i(a_{ix}, a_{iy})^T$ and $B_i(b_{ix}, b_{iy})^T$ are vectors of moving joints at position i . The mathematical model follows that in reference [37]. And the eight unknowns are $a_{0x}, a_{0y}, b_{0x}, b_{0y}, a_{1x}, a_{1y}, b_{1x}, b_{1y}$. In this paper, seven points are given, which is less than nine. We can acquire an infinite number of solutions in theory by solving the polynomial system, and all of the solutions can be mapped to the solution region. In this paper two unknowns must be given to realize the path synthesis. The open access software Bertini was utilized to solve the polynomial system [38].

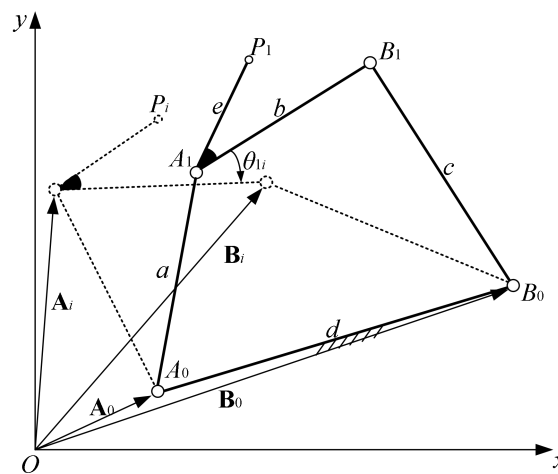


Fig. 1 Schematic of a hinge four-bar linkage from position 1 to position i

2.2 Obtaining valid real solutions

A large number of degeneration solutions will be obtained solving synthesis equations with Bertini. These solutions will generate links with dimension equal to zero, which does not exist in the actual. Therefore, these solutions should be screened out to retain valid real solutions before the solution region is established. The criteria are as follows:

$$a_{0x} = b_{0x} \ \& \ a_{0y} = b_{0y} \quad (1)$$

$$a_{1x} = b_{1x} \ \& \ a_{1y} = b_{1y} \quad (2)$$

$$a_{0x} = a_{1x} \ \& \ a_{0y} = a_{1y} \quad (3)$$

$$b_{0x} = b_{1x} \ \& \ b_{0y} = b_{1y} \quad (4)$$

When the linkage solution satisfies any of the above four discriminant conditions, it means that the solution belongs to the degeneration solution and needs to be removed from the solution set. All valid real solutions for establishing the solution region can be obtained through the above four discriminant conditions.

2.3 Defining the solution region plane

The purpose of establishing the solution region is to present all of the solutions in a two-dimensional plane visually, so as to make the linkage selection more convenient and efficient. In order to avoid the situation in Ref. [42] that two Roberts cognate linkages share one point in solution region and achieve that one point just corresponds to one linkage solution, x-coordinate and y-coordinate of the solution region of the solution region plane are defined as b_{1x} and b_{1y} respectively. Actually, points in the solution region are the joint B_1 of different linkages.

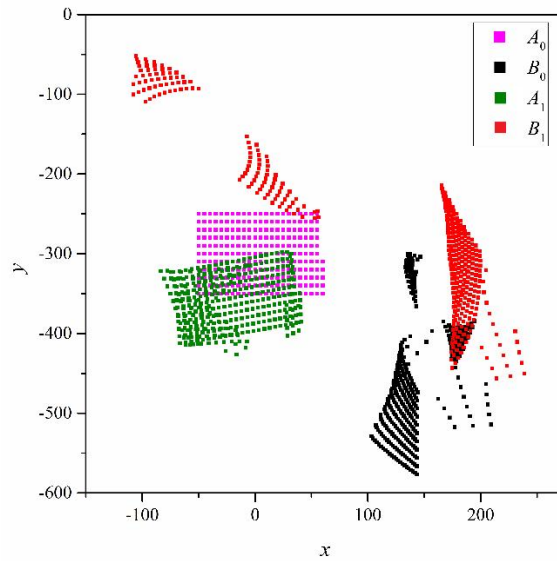


Fig. 2 The solution region of defect-free crank-rocker linkages established by joints A_0 , B_0 , A_1 , B_1 respectively

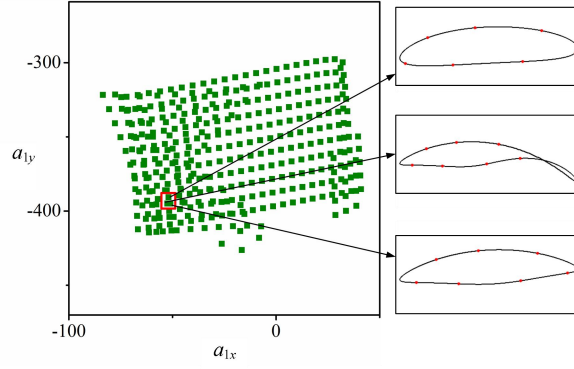


Fig. 3 Linkages of different curve types cluster together

Actually, each of joints A0, B0, A1, B1 can be as the reference to set up the solution region plane. Figure 2 is the solution region of defect-free crank-rocker linkages established by joints A0, B0, A1, B1 respectively. The following is the process of determining the solution region plane.

(1) In this paper, 426 defect-free crank-rocker linkages are obtained. In other words, each of the four solution regions should contain 426 points. But the number of pink points is much less than 426, which is 253. It reveals that some different linkage solutions map to the same point in the solution region in pink.

(2) Although there is no the problem that one point corresponds to multiple linkages in the green solution region, linkages of different curve types will cluster together as shown in Fig. 3, in other words, this solution region cannot be divided into different zones according to the curves types.

(3) Linkage solutions with different curve types distribute in different zones in the black solution region, but two Roberts cognates will share the same B0 (If the three pivots are A0, B0 and C0, one cognate locates A0 and B0, one cognate locates B0 and C0, the third locates A0 and C0.), which means that it will be a 2 to 1 map in the original solution region. This should be avoided.

(4) The problems arising in the black solution region and the green solution region do not occur in the solution region defined by B1. And linkage solutions with the same type of coupler curves cluster together. Finally, the solution region is established with B1 as the reference.

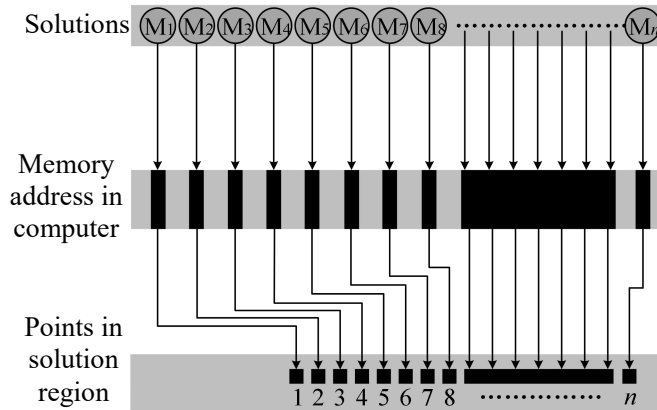


Fig. 4 The corresponding relation between linkages and points in the solution region built on the computer

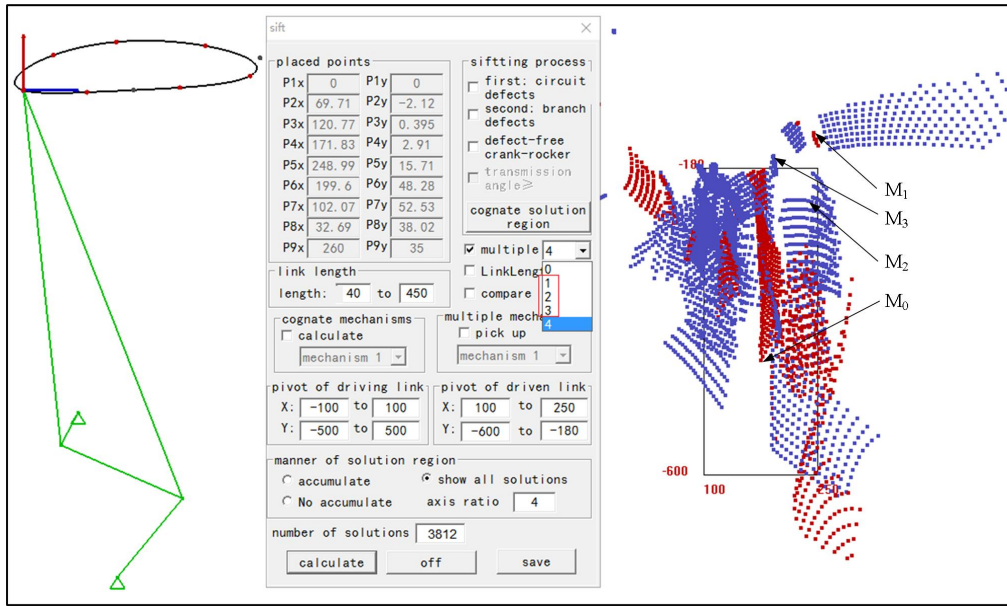


Fig. 5 Program interface after double clicking the linkage solution point M_0

The mapping relation between linkage solutions and points in the solution region can be easily built on the computer as shown in Fig. 4. Each point in the solution region carries one memory address for one linkage. Thus when one point in the solution region is selected, the corresponding linkage will be found and displayed on the computer screen. Figure 5 shows the screenshot of the program interface after double-clicking point M_0 . The left of Fig. 5 is the linkage corresponding to M_0 . And next to the linkage is “sift” dialog. The right of Fig.5 is the solution region built with B_1 after imposing the design requirements. And the other three forms of solution region can be displayed through selecting the option “1”, “2” or “3” in the red box. “1” corresponds to the solution region built with A_0 . Option “2” indicates that the program will present the solution region built with A_1 . And if the option “3” is selected, the solution region built with A_0 can be obtained. Figure 6 shows the three types of solution region corresponding to options “1”, “2” and “3” respectively.

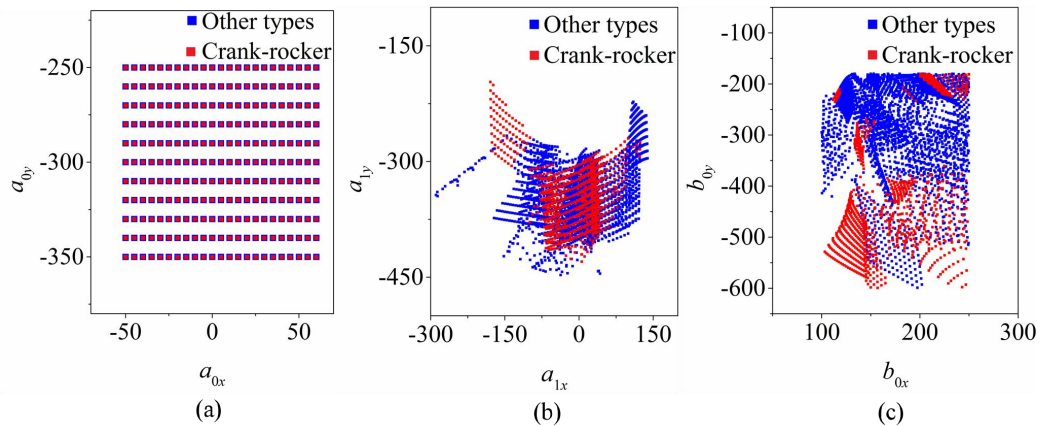


Fig. 6 The other three forms of solution region built with A_0 , A_1 and B_1 as the reference respectively: (a) The solution region built with A_0 as the reference corresponding to the option “1” in Fig. 5; (b) The solution region built with A_1 as the reference corresponding to the option “2” in Fig. 5; (c) The solution region built with B_0 as the reference corresponding to the option “3” in Fig. 5.

2.4 Defining the feasible solution region

Originally, the initial solution region contains all of the linkage solutions. There are a large number of linkages, the link lengths of which are too long and their dimensions are too large. While designers tend to acquire the linkages with compact dimensions and good mechanical performance.

So some design requirements should be given according to the actual working conditions to limit the dimensions and the fixed locations of linkages within an appropriate range. After imposing the design requirements, there are still some linkages that can only pass through a few of the seven given points in succession, and can only pass through the other given points if they are disassembled and reassembled. These linkages with circuit defects are not designers want and should be eliminated from the solution region. In addition, some linkages will get to the configuration with poor mechanical performance, the dead-center position, during passing through the given points, which is not fatal to the movement of linkages. So designers can determine whether to keep these linkages with branch defects in the feasible solution region according to the actual needs. So it is vital to determine which linkages should be remained in the feasible solution region and to identify and eliminate the useless linkages from the solution region. The following constraints are applied to the solution region to acquire the feasible solution region with only usable linkages.

(1) Imposing design requirements

The design requirements include fixed location constraints and dimensional constraints in this paper. In this study, the linkages satisfy the constraints following are remained in the solution region:

$$40 < l_a, l_b, l_c, l_d, l_e < 450 \quad (5)$$

$$100 < b_{0x} < 250 \quad (6)$$

$$-600 < b_{0y} < -180 \quad (7)$$

where l_a, l_b, l_c, l_d, l_e represent link lengths of link a, b, c, d, e respectively.

Imposing the design requirements, the number of solutions in the solution region decreases greatly. In this study, the number of solutions in the solution region is reduced from 23455 to 3812.

(2) Eliminating the defects

In this paper, circuit defects and branch defects are considered. All of the linkages with circuit defects should be filtered out from the solution region, while whether to remain the linkages with branch defects will depend on the designers. So filtering out circuit defects and branch defects are carried out in two steps on the computer program developed. Designers can obtain the feasible solution region, which remains the linkages with branch defects or not.

3. Defect judgment method

For four-bar linkage, Jacobian matrix determinant is traditionally used to determine whether the linkage has motion defects, but this method can not clearly determine linkages of type 2 in Table 1 has circuit defects or branch defects. Therefore, this paper adopts another simple defect judgment method to identify and eliminate the linkages with defects. Two vector parameters P_i and Q_i are defined to illustrate the rules of the defect judgment method for four-bar linkages as follows:

$$P_i = \overrightarrow{A_i B_0} \times \overrightarrow{A_i B_i} \quad (i=1, 2, 3, \dots, m) \quad (8)$$

$$Q_i = \overrightarrow{A_0 B_0} \times \overrightarrow{A_0 A_i} \quad (i=1, 2, 3, \dots, m) \quad (9)$$

where m is the number of the given points.

The rules of the defect judgment method is shown in table 1, where “ $P_i > 0$ ” and “ $Q_i > 0$ ” represent that their directions are coincident with z-axis and “ $P_i < 0$ ” and “ $Q_i < 0$ ” represent that their directions are opposite to the z-axis.

Table 1 Rules of defect judgment method for a four-bar linkage

Number	Linkage types	Circuit types	Branch types
1	Grashof crank-rocker linkage and Grashof double crank linkage	$\begin{cases} P_i > 0 (\text{OPEN}) \\ P_i < 0 (\text{CROSSED}) \end{cases}$	Only one branch
2	Grashof double rocker linkage and Grashof rocker-crank linkage	$\begin{cases} Q_i > 0 (\text{OPEN}) \\ Q_i < 0 (\text{CROSSED}) \end{cases}$	OPEN $\begin{cases} P_i \geq 0 (1) \\ P_i \leq 0 (2) \end{cases}$ CROSSED $\begin{cases} P_i \geq 0 (2) \\ P_i \leq 0 (1) \end{cases}$
3	Non-Grashof three rocker linkage	Only one circuit	$\begin{cases} P_i \geq 0 (1) \\ P_i \leq 0 (2) \end{cases}$

Here are three examples to illustrate how to filter out the defects, where link a is considered as the driver.

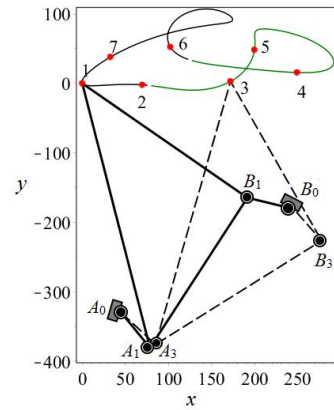
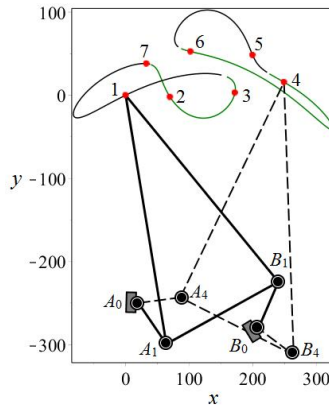
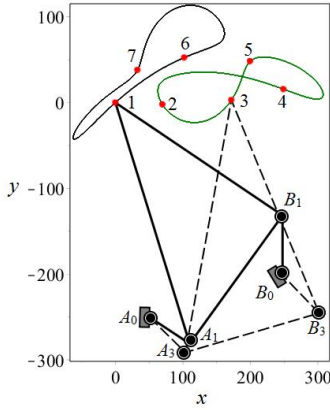


Fig. 7 Linkage solution point M₁

Fig. 8 Linkage solution point M₂

Fig. 9 Linkage solution point M₃

Number 1: a crank-rocker linkage corresponding to point M₁ in Fig. 5

As shown in Fig. 7, A₀A₁B₁B₀ and A₀A₃B₃B₀ represent the configurations at the given point 1 and the given point 3 respectively. We can get

$$K_1 = \overrightarrow{A_1B_0} \times \overrightarrow{A_1B_1} > 0 (\text{OPEN}) \quad (10)$$

$$K_3 = \overrightarrow{A_3B_0} \times \overrightarrow{A_3B_3} < 0 (\text{CROSSED}) \quad (11)$$

Eqs. (10) and (11) indicate that configuration A₀A₁B₁B₀ is in OPEN circuit and configuration A₀A₃B₃B₀ is in CROSSED circuit. Then the linkage will be filtered out.

Number 2: a rocker-crank linkage corresponding to point M₂ in Fig. 5

As shown in Fig. 8, A₀A₁B₁B₀ and A₀A₄B₄B₀ represent the configurations at the given point 1 and the given point 4 respectively. We can get

$$G_1 = \overrightarrow{A_0B_0} \times \overrightarrow{A_0A_1} < 0 (\text{CORSSSED}) \text{ and } K_1 = \overrightarrow{A_1B_0} \times \overrightarrow{A_1B_1} > 0 (2) \quad (12)$$

$$G_4 = \overrightarrow{A_0B_0} \times \overrightarrow{A_0A_4} > 0 (\text{OPEN}) \text{ and } K_4 = \overrightarrow{A_4B_0} \times \overrightarrow{A_4B_4} < 0 (2) \quad (13)$$

Eqs. (12) and (13) indicate that the linkage not only has circuit defects but also has branch defects. The linkage should be filtered out.

Number 3: a non-Grashof linkage corresponding to point M₃ in Fig. 5

A₀A₁B₁B₀ and A₀A₃B₃B₀ represent the configurations at the given point 1 and the given point 3 respectively as shown in Fig. 9. The green curve and the black curve represent two different branches. We can get

$$K_1 = \overrightarrow{A_1B_0} \times \overrightarrow{A_1B_1} > 0 (1) \quad (14)$$

$$K_3 = \overrightarrow{A_3B_0} \times \overrightarrow{A_3B_3} < 0 (2) \quad (15)$$

Eqs. (14) and (15) indicate that the linkage has branch defects and it will be decided whether to remain this linkage in the feasible solution region on the basis of actual needs.

4. A numerical example

Axis	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7
x	0	69.71	171.8 3	248.9 9	199.6	102.0 7	32.69
y	0	-2.12	2.91	15.71	48.28	52.53	38.02

Table 2 Positions of given points

This section gives seven points as shown in Table 2. The values of two unknowns must be given to solve the path synthesis problem with seven points. In this study the positions of A0 are given. If the step size of a_{0x} and a_{0y} is infinitesimal, an infinite number of solutions can be obtained. In order to facilitate the trials, proper step sizes should be set. Thus a finite number of solutions will be obtained. The locations of A0 are as follow:

$$a_{0x} = -50 + 5 \times j \quad (j=0, 1, 2, \dots, 22) \quad (12)$$

$$a_{0y} = -250 - 10 \times j \quad (j=0, 1, 2, \dots, 10) \quad (13)$$

23455 physical solutions that represent physical link geometry are obtained. With the developed computer program, the solution region is obtained. Figure 10 shows the solution regions after sifting by different constraint conditions. The X-axis represents the x-coordinate of B1 and the Y-axis represents the y-coordinate of B1. After applying the design requirements described in Eqs. (1), (2) and (3), 3812 linkages are retained in the solution region as shown in Fig. 10a. 2253 linkages can be acquired after eliminating the circuit defects, as shown in Fig. 10b. Finally, 476 linkages without defects are obtained as shown in Fig. 10c, where 426 of them are defect-free crank-rocker linkages. In general, the solution region shown in Fig. 10c is considered as the feasible solution region. But as stated in section 2.4, if the designer wants to remain the solutions with branch defects, the solution region shown in Fig. 10b can also be considered as the feasible solution region. Red points represent the crank-rocker linkages and the blue points represent the other types of linkages in the solution region.

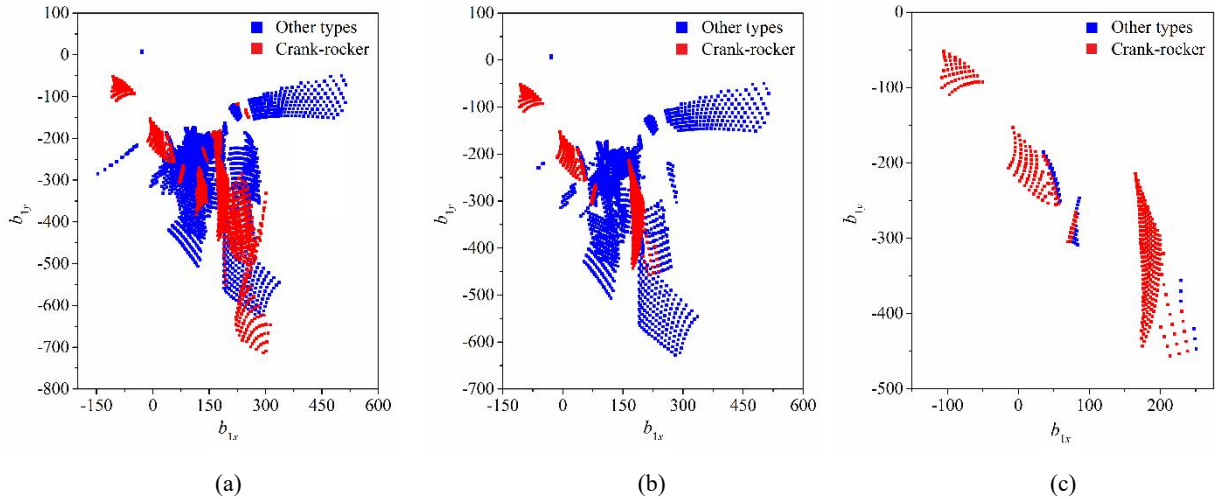


Fig. 10 Different solution regions after sifting by different constraint conditions: (a) the solution region after imposing design requirements, (b) the solution region without linkages with circuit defects, (c) the solution regions composed of the linkages without circuit defects and branch defects.

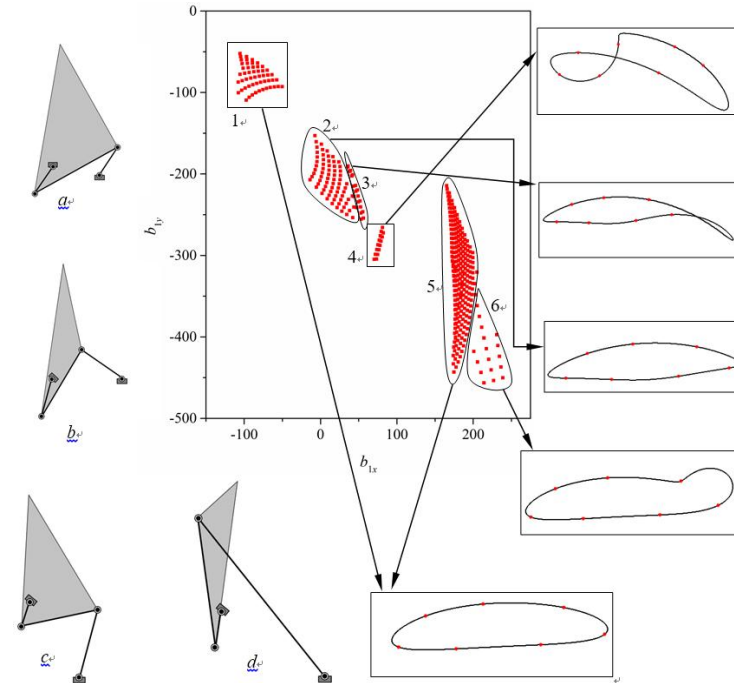


Fig. 11 Different areas in the solution region correspond to different types of coupler curves

Note that solution region just containing the defect-free crank-rocker linkages appears six zones. Different initial configurations and coupler curve types correspond to different zones as shown in Fig. 11. The types of coupler curves are more diverse than those in Ref. [36], where the curves corresponding to zones 2, 3 and 4 do not appear in Ref. [36]. Linkages in zone 1 are corresponding to initial configuration d. Linkages in zone 2 and zone 3 are corresponding to initial configuration b. Linkages in zone 4 are corresponding to initial configuration a. Linkages in zone 5 and zone 6 are corresponding to initial configuration c. The characteristic of the solution region presented in Fig. 11 is more helpful to quickly find crank-rocker linkages needed according to different curve types.

5. Conclusion

This paper applies SRS to seven-precision-point path synthesis for four-bar linkages. The key contribution of this paper is to give the method of establishing the solution region for multiple parameters. For eight-point path synthesis, just one variable needs to be given. No matter which one of joints A_0 , B_0 , A_1 , B_1 is used as the reference, there is no the situation that one point corresponds to multiple linkages in the feasible solution region. Since two parameters must be given, the number of linkage solutions in this case becomes more and the solution region becomes larger. The situation that one point corresponds to multiple linkages in the feasible solution region appears. And we need to define the solution region where linkage solutions with the same type of curves can get together. Comparing four different forms of solution regions, the solution region is finally defined with B_1 as the reference. The defect-free crank-rocker linkages appear in six different zones according to their curve types.

A numerical example is given in this paper. 476 defect-free linkages are obtained by giving 253 sets of (a_0x, a_0y) values. In other words, two defect-free linkages can be obtained for each set of values on average, which is twice the number of defect-free linkages obtained through eight-precision-point path synthesis. And the four-bar linkages obtained have three more curve types than the linkages obtained through eight-precision-point path synthesis. With the developed computer program, the visualization of solution region and linkages can be realized. The feasible solutions can be obtained quickly using the developed computer program.

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