

# Location and Distribution Decision of Emergency Logistics Network under Uncertain Demand Distributions

Ziqi Liu<sup>1, a</sup>, Fei Wen<sup>2, b</sup>, Dali Zhang<sup>1, c \*</sup>, Shuang Hao<sup>3, d</sup>

<sup>1</sup> Sino-US Global Logistics Institute, Shanghai Jiao Tong University, Shanghai, China;

<sup>2</sup> School of Electronic Information and Electrical Engineering, Shanghai Jiao Tong University, Shanghai, China;

<sup>3</sup> Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, China.

<sup>a</sup> lzq0603@sjtu.edu.cn, <sup>b</sup> wenfei@sjtu.edu.cn, <sup>c \*</sup> zhangdl@sjtu.edu.cn, <sup>d</sup> sjtu-haos@sjtu.edu.cn

**Abstract.** To ensure the supply of materials after urgent events, including recent public health crisis, it is crucial to formulate a reasonable preventive strategy for the location and distribution of emergency materials within the logistics network. In this paper, a class of multi-category emergency material storage distribution models under scenarios with uncertain post-event demand distribution is studied. Based on historical small-sample post-event demand information, the model constructs a two-stage material storage and scheduling distributionally robust optimization model. By applying a dual method, the non-linear distributionally robust optimization is transformed into linear optimization, and an improved Lagrangian L-shaped algorithm is designed to solve the two-stage model. Finally, the robustness of the model and algorithm is verified by implementing the method to a numerical example, and the sensitivity of the location and distribution decisions to material shortage under different levels of urgent events is analyzed.

**Keywords:** Emergency planning; Uncertain scenarios; Distributionally robust optimization; Two-stage model; Lagrangian L-shaped algorithm.

## 1. Introduction

In recent years, sudden events such as the COVID-19 pandemic have occurred frequently, always accompanied by urgent demands for various living and medical emergency materials. Therefore, how to use historical small samples to achieve preventive storage selection and material distribution strategies and reasonably carry out material allocation has always been a difficult issue in related research.

Current research on emergency material allocation and scheduling mainly falls into two categories. One focuses on quickly satisfying demands after emergencies to minimize losses<sup>[1, 2]</sup>; the other considers how to minimize the cost of pre-event material reserves and maintenance under the condition of effectively meeting post-disaster demands<sup>[3-5]</sup>. Both research areas face the problem of post-disaster demand uncertainty, including factors such as demand quantity, location distribution, and material transportation network structure<sup>[6-8]</sup>. In response to the uncertainty in emergency material allocation, Balcik et al.<sup>[9]</sup> first proposed establishing demand through historical data samples and maximizing the coverage of sudden demands with the stochastic coverage model. CONG and YU<sup>[10]</sup> divided typhoon paths into different scenarios through the mean clustering algorithm, studied the material demands corresponding to various scenarios. In terms of time uncertainty, Rawls and Turnquist<sup>[11]</sup> studied the material allocation problem before the emergency event and the scheduling problem under various possible scenarios after the event. On the other hand, Hamdan and Diabat<sup>[12]</sup> focused on the blood demand after sudden public health events under given possible scenarios, establishing a blood reserve and supply network model with the goal of minimizing expected cost and response time. Recent research pays more attention to how to design storage strategies to balance the overall cost under various demand distributions and the supply capacity under extreme scenarios. Mulvey et al.<sup>[13]</sup> and Aghezzaf et al.<sup>[14]</sup> proposed distributionally robust optimization models for warehouse network distribution and material scheduling under the worst demand distribution and maximum demand difference scenarios, respectively. In subsequent

research, Jabbarzadeh et al.<sup>[15]</sup> built a medical material supply location and post-event allocation strategy based on the former, and Salehi et al.<sup>[16]</sup> designed a multi-period material storage network optimization model considering the substitutability between materials.

In summary, emergency events have a low probability of occurrence, and for specific categories of emergency events, only very limited sample data can be obtained, so the demand probability fitting after the occurrence of specific emergency events also has a large deviation <sup>[17, 18]</sup>. This study will address this issue, based on the statistical characteristics of limited samples, to establish a set of all possible demand distributions and a distributionally robust optimization model on this set. This paper applies a Lagrangian L-shaped method to effectively solve the above model.

## 2. The Two-Stage Distributionally Robust Optimization Model

### 2.1 Problem Description

This paper considers an emergency network composed of emergency material warehouses, material demand points, and transfer nodes, and establishes a model and distributionally robust optimization model for emergency material storage networks under uncertain scenarios. The decisions that need to be made before the emergency demand occurs include: emergency material warehouse location selection and warehouse pre-storage quantities. After the emergency event, based on the determined warehouse location selection and pre-storage strategy, continue to make the optimal decision for the transportation flow under specific demand scenarios.

Set the following parameters for the model.  $(N, A)$  is the storage network, where  $N$  is the set of nodes  $i$ ,  $A$  is the set of arcs  $(i, j)$ ,  $\forall i, j \in N$ .  $L$  is the set of warehouse levels  $l$ .  $K$  is the set of emergency material types  $k$ .  $S$  is the set of scenarios after the sudden event occurs  $s$ ,  $s \in S = \{1, 2, \dots, s, \dots\}$ .  $F_l$  is the fixed cost of establishing a level  $l$  warehouse.  $B_l$  is the standard capacity of a level  $l$  warehouse.  $b^k$  is the unit space of material  $k$ .  $h^k$  is the unit inventory holding cost of material  $k$ .  $\tau^k$  is the unit flow conversion coefficient of material  $k$ .  $p_i^k$  is the unit pre-storage cost of material  $k$  at the node  $i$ .  $c_{ij}^k$  is the unit transportation cost of material  $k$ .  $\tau_s$  is the loss caused by the sudden event scenario  $s$ .  $E^-(\tau)$  is the expected loss lower bound of the sudden event scenario.  $E^+(\tau)$  is the expected loss upper bound of the sudden event scenario.  $v_i^{ks}$  and  $\rho_i^{ks}$  indicates the demand and the supply rate of material  $k$  at the node  $i$  when the scenario  $s$  occurs.  $q^{ks}$  is the unit shortage cost of material  $k$  when the scenario  $s$  occurs.  $\eta_{ij}^s$  is the pass rate of arc  $(i, j)$  when the scenario  $s$  occurs,  $\eta_{ij}^s \in \{0, 1\}$ ,  $\eta_{ij}^s = 1$  indicates that the arc  $(i, j)$  is passable, otherwise impassable.  $U_{ij}^s$  is the maximum transport capacity of arc when the scenario  $s$  occurs.

The decision variables included in the model are as follows. The first stage of the emergency event-related location decision  $y_{il}$  and pre-storage decision  $r_i^k$  before the emergency event.  $y_{il} \in \{0, 1\}$ ,  $y_{il} = 1$  indicates the need to establish a level  $l$  warehouse at node  $i$ .  $r_i^k$  indicates the pre-storage quantities of material  $k$  at node  $i$ . The second stage of the emergency event-related scheduling decision:  $P_s$  is the probability of scenario  $s$  occurrence, the occurrence of uncertain scenarios is difficult to determine, the random variable  $P_s$  obeys the distribution under the given sudden event scenario occurrence condition constraint.  $x_{ij}^{ks}$  is the transportation quantities of material  $k$  on arc  $(i, j)$  when the scenario  $s$  occurs.  $u_i^{ks}$  and  $w_i^{ks}$  indicates the remaining inventory and the shortage quantities of material  $k$  at node  $i$  when the scenario  $s$  occurs.

## 2.2 Model

The warehouse network and material distribution model for emergency demand is as follows, including the main problem model P1 before the emergency event in the first stage, where  $H(\mathbf{y}, \mathbf{r})$  is the expected cost after the worst demand distribution generated by the decision-making in the second stage following the warehouse location selection  $\mathbf{y} = \{y_{il} : \forall i \in N, l \in L\}$  and pre-storage decision  $\mathbf{r} = \{r_i^k : \forall k \in K, i \in N\}$  given in the first stage, and the specific expression is given by P2.

$$(P1) \min \sum_{i \in N} \sum_{l \in L} F_l y_{il} + \sum_{i \in N} \sum_{k \in K} p_i^k r_i^k + H(\mathbf{y}, \mathbf{r}) \quad (1)$$

$$\text{s.t.} \quad \sum_{l \in L} y_{il} \leq 1, \forall i \in N \quad (2)$$

$$\sum_{k \in K} b^k r_i^k \leq \sum_{l \in L} B_l y_{il}, \forall i \in N \quad (3)$$

$$y_{il} \in \{0, 1\}, \forall i \in N, l \in L \quad (4)$$

$$r_i^k \geq 0, \forall k \in K, i \in N \quad (5)$$

Among them, equation (1) is the objective function of the first stage, which aims to minimize the comprehensive cost of warehouse location selection, emergency material pre-storage procurement cost, and the expected cost brought about by the worst demand distribution in the second stage after the location and pre-storage decision are determined; equation (2) is the warehouse location selection constraint, that is, the number of supply points selected at each node shall not exceed 1; equation (3) is the warehouse capacity constraint, that is, the pre-stored material quantities shall not exceed the corresponding capacity of the supply warehouse; equations (4-5) are decision variable constraints.

Based on the warehouse location selection decision and pre-storage decision of P1, the sub-problem model P2 in the second stage is as follows:

$$(P2) H(\mathbf{y}, \mathbf{r}) = \min_{\mathbf{x}, \mathbf{u}, \mathbf{w}} \max_{\mathbf{P}} f(\mathbf{x}, \mathbf{u}, \mathbf{w}, \mathbf{P}) \quad (6)$$

$$f(\mathbf{x}, \mathbf{u}, \mathbf{w}, \mathbf{P}) = \sum_{s \in S} P_s \left[ \sum_{(i,j) \in A} \sum_{k \in K} c_{ij}^k x_{ij}^{ks} + \sum_{i \in N} \sum_{k \in K} (h^k u_i^{ks} + q^{ks} w_i^{ks}) \right] \quad (7)$$

$$\text{s.t.} \quad \left[ \sum_{j \neq i \in N} x_{ji}^{ks} + \rho_i^{ks} r_i^k \right] - \left[ \sum_{j \neq i \in N} x_{ij}^{ks} + v_i^{ks} \right] = u_i^{ks} - w_i^{ks}, \forall s \in S, k \in K, i \in N \quad (8)$$

$$\sum_{k \in K} \eta_{ij}^s x_{ij}^{ks} \leq U_{ij}^s, \forall s \in S, (i, j) \in A \quad (9)$$

$$x_{ij}^{ks} \geq 0, \forall s \in S, k \in K, (i, j) \in A \quad (10)$$

$$u_i^{ks}, w_i^{ks} \geq 0, \forall s \in S, k \in K, i \in N \quad (11)$$

$$P_s \geq 0, \forall s \in S \quad (12)$$

Among them, equation (6) is the objective function of the second stage, where  $\mathbf{P}$  is the probability distribution function  $\mathbf{P} = \{P_s, \forall s \in S\}$  and  $\mathbf{P}$  is the set of all possible probability distribution functions that satisfy the historical data sample, that is

$$\mathbf{P} = \left\{ \mathbf{P} \left| \begin{array}{l} P_s \geq 0, \forall s \in S \\ \sum_{s \in S} P_s = 1 \\ \sum_{s \in S} P_s \tau_s \geq E^-(\tau) \\ \sum_{s \in S} P_s \tau_s \leq E^+(\tau) \end{array} \right. \right\} \quad (13)$$

$P_s \geq 0, \sum_{s \in S} P_s = 1$  ensures that  $\mathbf{P}$  is a probability distribution.  $\sum_{s \in S} P_s \tau_s \geq E^-(\tau)$  and  $\sum_{s \in S} P_s \tau_s \leq E^+(\tau)$

are the minimum and maximum expected losses after the sudden event occurs. In addition,  $\mathbf{x}, \mathbf{u}, \mathbf{w}$  in the objective function  $f(\mathbf{x}, \mathbf{u}, \mathbf{w}, \mathbf{P})$  represents the decision variables that need to be made in the second stage model, including:

Transportation variables  $\mathbf{x} = \{x_{ij}^{ks}, \forall s \in S, (i, j) \in A, k \in K\}$ ,

Inventory variables  $\mathbf{u} = \{u_i^{ks}, \forall s \in S, i \in N, k \in K\}$ ,

Shortage variables  $\mathbf{w} = \{w_i^{ks}, \forall s \in S, i \in N, k \in K\}$ .

Here, Equations (8)-(12) are the constraints, where Equation (8) is the network node flow balance constraint. The sum of the inflows at each node minus the sum of the outflows, the pre-storage quantities, and the material demand, if the difference is positive, then there will be surplus materials at this node, otherwise, a negative value indicates a material shortage. Equation (9) is the network arc capacity constraint after the sudden event occurs. Equations (10)-(12) are the decision variable constraints.

### 2.3 The Model Duality

Due to the existence of the set  $P$ , the probability  $P_s$  of the scenario  $s$  occurring in the decision-making process after the disaster is uncertain. The decision-making process of the P2 model needs to consider the worst result among all probability distributions  $\mathbf{P} = \{P_s, \forall s \in S\}$  that satisfy the set  $P$  condition, thus  $P_s$  it is also one of the decision variables in this distributionally robust optimization problem. As a result, the objective function of the P2 model includes the product of decision variables  $P_s$  and other variables, which brings certain difficulties to the model solution.

Here, we will establish a framework for dual transformation to convert the problem into a linear model. First, the set is represented in matrix form (14), where  $\bar{A}$  represents the constraint coefficient matrix,  $\bar{A} = [a_1, \dots, a_s], a_s = [1, -1, -\tau_s, \tau_s]^T, \forall s \in S$ ,  $\bar{b}$  represents the constraint term,  $\bar{b} = [1, -1, -E^-(\tau), E^+(\tau)]^T$ ,  $\bar{P}$  represents the random probability variable term,  $\bar{P} = [P_1, \dots, P_s]^T, \forall s \in S$ .

$$\bar{A} \cdot \bar{P} \leq \bar{b} \quad (14)$$

Given the second-stage decision  $\mathbf{x}, \mathbf{u}, \mathbf{w}$ , the objective function (6) and the set of probability distribution functions (14) are dual-transformed as follows, and the dual variables for the set of emergency event scenario probability distribution functions are set as  $\Gamma = \{\zeta_d, \forall d \in D\}$ , and  $D = \{1, 2, 3\}$  is the set of constraint numbers in the probability distribution function.

$$\min_{\Gamma} h(\Gamma) \quad (15)$$

$$h(\Gamma) = \bar{b}^T \cdot \Gamma \quad (16)$$

$$\sum_{d \in D} a_s \zeta_d \geq \sum_{(i,j) \in A} \sum_{k \in K} c_{ij}^k x_{ij}^{ks} + \sum_{i \in N} \sum_{k \in K} (h^k u_i^{ks} + q^{ks} w_i^{ks}), \forall s \in S \quad (17)$$

After the dual transformation, the objective function is transformed into equation (15), and the constraints are transformed into equation (17). The decision variables in the original objective function are converted to equation (17), while also satisfying the second-stage model constraints (8)-(11). After the above dual transformation, the second-stage non-linear robust optimization model P2 is transformed into a linear optimization model P3, and constraints (8)-(11) remain unchanged:

$$(P3) H(y, r) = \min_{\Gamma} h(\Gamma) \quad (18)$$

$$h(\Gamma) = \bar{b}^T \cdot \Gamma \quad (19)$$

$$\text{s.t. } \sum_{d \in D} a_s \zeta_d \geq \sum_{(i,j) \in A} \sum_{k \in K} c_{ij}^k x_{ij}^{ks} + \sum_{i \in N} \sum_{k \in K} (h^k u_i^{ks} + q^{ks} w_i^{ks}), \forall s \in S \quad (20)$$

$$\zeta_d \geq 0, \forall d \in D \quad (21)$$

### 3. Solution Method

#### 3.1 Model Decomposition

This study will solve the non-linear problem in the optimization objective of P3 based on the L-shaped method proposed by Van Slyke and Wets<sup>[19]</sup>. In our algorithm, a set of approximate functions are used to externalize the linearization of material pre-storage quantities  $r_i^k$ . The approximate functions utilize  $H(y, r)$  the partial derivatives of  $r_i^k$  changes to set linear approximation intercept terms  $E_t$  and slope coefficients  $e_{it}^k, \forall t \in T$ ,  $T$  representing the number of external linear approximation constraints. Here,  $\gamma_i^s$  represents the dual variable of the flow balance constraint (8) in P3, and the calculations for  $E_t$  and  $e_{it}^k$  are as follows:

$$e_{it}^k = \sum_{s \in S} \sum_{k \in K} P_s \gamma_i^{ks}, \forall k \in K, i \in N, t \in T \quad (22)$$

$$E_t = \sum_{s \in S} \sum_{i \in N} \sum_{k \in K} P_s \gamma_i^{ks} v_i^{ks}, \forall t \in T \quad (23)$$

By introducing intermediate variable  $\theta$  approximations  $H(y, r)$ , P1 is transformed into the following P4 model, and constraints (2)-(5) remain unchanged:

$$(P4) \min \sum_{i \in N} \sum_{l \in L} F_l y_{il} + \sum_{i \in N} \sum_{k \in K} p_i^k r_i^k + \theta \quad (24)$$

$$\theta \geq E_t - \sum_{i \in N} \sum_{k \in K} e_{it}^k r_i^k, t \in T \quad (25)$$

Here, each time P4 is solved, the current solution can be checked via the optimal value  $H(y, r)$  of P3, that is, if during the iteration process, then the optimality  $\theta = H(y, r)$  is achieved; otherwise  $\theta < H(y, r)$ , a new optimal cut is added, transformed into a semi-infinite constraint (25) and included in the iterative solution of P4.

For the first-stage P4 problem, the Lagrangian relaxation method is further used to decompose the warehouse location integer programming model and the pre-storage linear programming model for solution<sup>[20]</sup>. That is, the warehouse capacity constraint (3) in P4 is relaxed and transformed into the objective function (26), and the Lagrangian multiplier is set as  $\mu_i, \forall i \in N$ . The objective function in the P4 problem is:

$$\min \sum_{i \in N} \sum_{l \in L} F_l y_{il} + \sum_{i \in N} \sum_{k \in K} p_i^k r_i^k + \theta + \sum_{i \in N} \mu_i \left( \sum_{k \in K} b^k r_i^k - \sum_{l \in L} B_l y_{il} \right) \quad (26)$$

By splitting the location variables  $y_{il}$  and pre-storage quantities  $r_i^k$  in the above formula, P4 can be decomposed into the location integer programming model P4a related to  $y_{il}$  and the material pre-storage linear programming model P4b related to  $r_i^k$ , as follows:

$$(P4a) \min \sum_{i \in N} \sum_{l \in L} (F_l - \mu_i B_l) y_{il} \quad (27)$$

$$\text{s.t. } \sum_{l \in L} y_{il} \leq 1, \forall i \in N \quad (28)$$

$$y_{il} \in \{0,1\}, \forall i \in N, l \in L \quad (29)$$

$$(P4b) \min \sum_{i \in N} \sum_{k \in K} (p_i^k + \mu_i b^k) r_i^k \quad (30)$$

$$\text{s.t. } \theta \geq E_t - \sum_{i \in N} \sum_{k \in K} e_{it}^k r_i^k, t \in T \quad (31)$$

$$r_i^k \geq 0, \forall i \in N \quad (32)$$

For the warehouse location integer programming model P4a, a greedy algorithm is used for solution: for any node  $i$ , if the minimum check number  $\min_l (F_l - \mu_i B_l)$  is less than 0, then set up the corresponding supply warehouse type  $l$ , i.e.,  $y_{il} = 1$ , otherwise set  $y_{il} = 0$ . P4b is a standard linear programming model that, combined with the decision results of P3, determines the optimal pre-storage quantity for different emergency materials.

### 3.2 LLSM Algorithm

The specific algorithm is as follows. First, define the parameters  $T$  to represent the number of optimal partitioning constraints (25) in the model P4;  $\delta$  represents the number of Lagrangian multiplier iterations;  $\mu_i^\delta, \forall i \in N$  represents the Lagrangian multiplier at the  $\delta$  th iteration;  $\phi^*$  represents the current optimal value of the two-stage model (i.e., the main problem model P4), initialized to infinity;  $\varphi_{P4a}, \varphi_{P4b}$  represents the optimal value of the warehouse location model P4a and the material pre-storage model P4b;  $\mathbf{y}(T), \mathbf{r}(T), \theta(T)$  represents the optimal solution of the main problem model P4 under  $T$  optimal partitioning constraints;  $\mathbf{r}^\delta(T), \theta^\delta(T)$  represents the optimal solution and feasible solution of the warehouse location model P4a at the  $\delta$  th iteration;  $\mathbf{r}^\delta(T), \theta^\delta(T)$  represents the optimal solution and external linear approximation variable optimal solution of the material pre-storage model P4b at the  $\delta$  th iteration;  $upperB, lowerB$  represents the upper and lower bounds of the main problem model P4;  $\varepsilon$  represents the deviation of the upper and lower bounds of model P4. The specific steps of the Lagrangian L-shaped algorithm (LLSM) are as follows:

Step 0. Parameter initialization. Set  $T = \delta = 0$ ;  $\mu_i^\delta = 0, \forall i \in N$ ;  $\phi^* = \infty$ .

Step 1. Solve the inner main problem using the Lagrangian multiplier iterative method. Judge whether  $T$  is equal to 0, if so, there are no optimal partitioning constraints at this time, set  $\theta^\delta(T) = 0$ , otherwise set  $\delta \leftarrow \delta + 1$ , according to the current Lagrangian multiplier value  $\mu_i^\delta$ , solve models P4a and P4b to obtain the optimal solution  $\mathbf{y}^\delta(T), \mathbf{r}^\delta(T), \theta^\delta(T)$  and optimal value  $\varphi_{P4a}, \varphi_{P4b}$ .

Step 2. Update the upper and lower bounds of the inner main problem. Among them, the lower bound of the main problem model P4 is  $lowerB = \varphi_{P4a} + \varphi_{P4b}$ ; the optimal solution  $\mathbf{y}^\delta(T)$  of the relaxed model P4a obtains a feasible solution  $\bar{\mathbf{y}}^\delta(T)$ , making the optimal solution  $\mathbf{r}^\delta(T)$  of model P4b have a supply warehouse at the pre-storage node in the logistics network and not exceeding the warehouse capacity, and calculate the upper bound  $upperB = \min \{ \phi^*, \varphi_{P4a} [\bar{\mathbf{y}}^\delta(T)] + \varphi_{P4b} \}$  of the main problem model P4.

Step 3. Judge whether the inner main problem converges to the optimal solution. Calculate the deviation of the upper and lower bounds  $\varepsilon = \frac{upperB - lowerB}{upperB}$  of model P4, if the deviation  $\varepsilon$  is

less than the iteration stopping threshold, then the Lagrangian relaxation method has converged to the approximate solution of model P4, update the current optimal solution of the inner main

problem,  $y(T) \leftarrow \bar{y}^\delta(T), r(T) \leftarrow r^\delta(T), \theta(T) \leftarrow \theta^\delta(T)$ , and transfer to step 5; otherwise, transfer to step 4.

Step 4. Update the Lagrangian multiplier using the subgradient search. Update the multiplier as in equation (41), and transfer to step 1.

$$\mu_i^{\delta+1} = \mu_i^\delta + \frac{1}{\delta} \left[ \sum_{k \in K} b^k r_i^{k\delta}(T) - \sum_{l \in L} B_l y_{il}^\delta(T) \right] \quad (33)$$

Step 5. Solve the outer two-stage model using the L-shaped method. Based on the optimal solution of the first-stage main problem, solve the second-stage sub-problem model to obtain the optimal value  $H[y(T), r(T)]$ , update the current optimal value of the first-stage main problem:

$$\phi^* = \sum_{i \in N} \sum_{l \in L} F_l y_{il}(T) + \sum_{i \in N} \sum_{k \in K} p_i^k r_i^k(T) + H[y(T), r(T)] \quad (34)$$

If  $H[y(T), r(T)] = \theta(T)$ , then the optimal solution is obtained, the algorithm stops; otherwise, calculate the parameters  $E_T$  and  $e_{iT}^k$ , add new optimal partitioning constraints to model P4, reset the parameters  $T \leftarrow T + 1$ ;  $\delta = 0$ ;  $\mu_i^\delta = 0, \forall i \in N$ , and transfer to step 1.

#### 4. Numerical Analysis

To verify the effectiveness of the model and algorithm in this study, a post-disaster local area emergency material supply case is established based on actual data, including a fully connected emergency material supply network with 10 nodes, 5 possible sudden events, and focusing on the supply of 3 major types of materials. The 10 nodes for potential location selection are distributed in conjunction with the administrative divisions of Shanghai. The transportation distances (kilometers) are calculated based on the actual location.

This paper continues the classification of emergency materials into three categories as studied by Rawls and Turnquist<sup>[11]</sup>, namely water resources (unit: thousands of liters), food resources (unit: thousands of instant food), and medical materials. Medical materials are considered as a collection of medical items, and the cost and space estimates for obtaining, storing, and transporting these goods are as shown in Table 1.

Table 1. Parameters of three kinds of emergency materials.

Types of emergency materials	The unit pre-storage cost (yuan/unit)	The unit space (cubic meters/unit)	The unit transportation cost (yuan/unit-kilometer)
Water resources (kiloliter)	4533.90	1012.20	2.10
Food resources (kilopiece)	37940.00	583.31	0.28
Medical materials (piece)	980.00	8.12	4.06E-3

Based on the unit purchase cost information of emergency materials, it is assumed that the pre-storage unit cost of each type of emergency material is the unit purchase cost multiplied by a related coefficient, which is related to the loss incurred by nodes in uncertain scenarios, set as  $\prod_{s \in S} \log(\tau_s)$ ;

the unit penalty cost for unmet material demand is the unit purchase price multiplied by a certain multiple  $\tau_s$ , and the unit inventory remaining cost is 25% of the purchase price. At the same time, the case considers three possible sizes of emergency material supply warehouse facilities, with parameters as shown in Table 2. Facilities of any size can be opened at any node in the network.

Table 2. Parameters of three kinds of warehouse facilities.

Warehouse levels	The fixed cost of location (ten thousand yuan)	The standard capacity (cubic meter)
------------------	--	-------------------------------------

Small	83.72	36400
Medium	131.88	408200
Large	210	780000

Table 3 lists the characteristics of emergency scenario 1, including the event location, demand for three types of emergency materials, designated roads impassable, and the loss assessed. The expected loss upper and lower bound interval for uncertain sudden event scenarios is set as the standard deviation range of all scenario losses, i.e., [8.63, 18.37].

Table 3. Parameters of three kinds of possible scenarios of emergencies.

Scenario id	Node id	Impassable roads	Demand			Loss (Hundred million yuan)
			water resources	Food resources	Medical materials	
1	1	(1,2)	350	525	500	10
1	3	(2,3)	350	525	500	15

The Lagrangian L-shape algorithm in this paper is implemented in Python, and the program is tested and run on a computer with 8 cores and 16GB of memory. For the linear programming model in the algorithm, Gurobi 10.0.1 is applied for solution. The solution results of the LLSM algorithm for the example show that the total cost of supply warehouse location decision in the first stage model decision optimization result is 837,200 yuan, and the total procurement cost of emergency material pre-storage is 50,579,900 yuan. A small warehouse is set up at node 8 of the network, and this supply warehouse pre-stores 900 units of emergency water resources, 1,200 units of food resources, and 996 units of medical resources. Based on the location and material storage decision results of the first stage model, different material scheduling plans are adopted for the possible scenarios 1 to 5 of uncertain sudden events, and the corresponding scheduling cost results are shown in Table 4.

Table 4. Scheduling costs under uncertain emergency scenarios.

Scenario id	Transportation cost	Holding cost	Shortage cost	All cost
1	3.72	164.94	980.00	1148.67
2	0.14	1143.73	0.00	1143.87
3	0.77	807.77	0.00	808.54
4	0.89	921.58	0.00	922.47
5	4.09	2.35	0.00	6.44

## 5. Summary

This paper has studied the location and distribution decisions of the emergency logistics network under uncertain demand distributions. By using the probability distribution characteristics based on historical data, a set of possible demand distributions is established. This set imposes constraints on possible probability distributions, and a dual transformation is applied to linearize the non-linear distributionally robust optimization. An improved Lagrangian L-shaped algorithm is designed to solve the two-stage model by dividing it into an inner and outer loop, and the effectiveness of the model and algorithm is verified using an actual case study. Future research could consider expanding this algorithmic framework for more complex real-world scenarios.

## Acknowledgements

This work was supported by the STI 2030—Major Projects (2022ZD0208700).



## References

- [1] HE Yuanyuan, JIANG Pansong, WEN Luxing, et al. Distribution Strategy of Multiple Epidemic Prevention Materials Weighing Fairness, Economy and Efficiency[J]. Engineering and Management, 2021, 26(6): 146-153.
- [2] Zhou Q S, Olsen T L. Inventory rotation of medical supplies for emergency response[J]. European Journal of Operational Research, 2017, 257(3): 810-821.
- [3] Santoso T, Ahmed S, Goetschalckx M, et al. A stochastic programming approach for supply chain network design under uncertainty[J]. European Journal of Operational Research, 2005, 167(1): 96-115
- [4] Wang X (Jocelyn), Paul J A. Robust optimization for hurricane preparedness[J]. International Journal of Production Economics, 2020, 221: 107464.
- [5] Kelle P, Schneider H, Yi H. Decision alternatives between expected cost minimization and worst case scenario in emergency supply – Second revision[J]. International Journal of Production Economics, 2014, 157: 250-260.
- [6] ZHANG Ling, YE Xianbao, CHEN Shengqun. Stochastic Optimal Resource Allocation Model and Algorithm Based on Linear Rule[J]. Journal of Systems Science and Mathematical, 2017, 37(5): 1221-1230.
- [7] Hoyos M C, Morales R S, Akhavan-Tabatabaei R. OR models with stochastic components in disaster operations management: A literature survey[J]. Computers & Industrial Engineering, 2015, 82: 183-197.
- [8] Haghani A, Oh S C. Formulation and solution of a multi-commodity, multi-modal network flow model for disaster relief operations[J]. Transportation Research Part A: Policy and Practice, 1996, 30(3): 231-250.
- [9] Balci B, Beamon B M, Smilowitz K. Last Mile Distribution in Humanitarian Relief[J]. Journal of Intelligent Transportation Systems, 2008, 12(2): 51-63.
- [10] CONG Wenjing, YU Wuyang. Model and Algorithm for Location of Regional Emergency Material Reserve Site in Typhoon Scenarios[J]. Industrial Engineering and Management, 2020, 25(5): 68-74.
- [11] Rawls C G, Turnquist M A. Pre-positioning of emergency supplies for disaster response[J]. Transportation Research Part B: Methodological, 2010, 44(4): 521-534.
- [12] Hamdan B, Diabat A. Robust design of blood supply chains under risk of disruptions using Lagrangian relaxation[J]. Transportation Research Part E: Logistics and Transportation Review, 2020, 134: 101764.
- [13] Mulvey J M, Vanderbei R J, Zenios S A. Robust Optimization of Large-Scale Systems[J]. Operations Research, 1995, 43(2): 264-281.
- [14] Aghezzaf E H, Sitompul C, Najid N M. Models for robust tactical planning in multi-stage production systems with uncertain demands[J]. Computers & Operations Research, 2010, 37(5): 880-889.
- [15] Jabbarzadeh A, Fahimnia B, Seuring S. Dynamic supply chain network design for the supply of blood in disasters: A robust model with real world application[J]. Transportation Research Part E: Logistics and Transportation Review, 2014, 70: 225-244.
- [16] Salehi F, Mahootchi M, Hussein S M M. Developing a robust stochastic model for designing a blood supply chain network in a crisis: a possible earthquake in Tehran[J]. Annals of Operations Research, 2019, 283(1-2): 679-703.
- [17] Snyder L V. Facility location under uncertainty: a review[J]. IIE Transactions, 2006, 38(7): 547-564.
- [18] Gabrel V, Lacroix M, Murat C, et al. Robust location transportation problems under uncertain demands[J]. Discrete Applied Mathematics, 2014, 164: 100-111.
- [19] Van Slyke R M, Wets R. L-Shaped Linear Programs with Applications to Optimal Control and Stochastic Programming[J]. SIAM Journal on Applied Mathematics, 1969, 17(4): 26.
- [20] Fisher M L. An Applications Oriented Guide to Lagrangian Relaxation[J]. Interfaces, 1985, 15(2): 10-21.